Exam 1.

1. Definition of plasma:

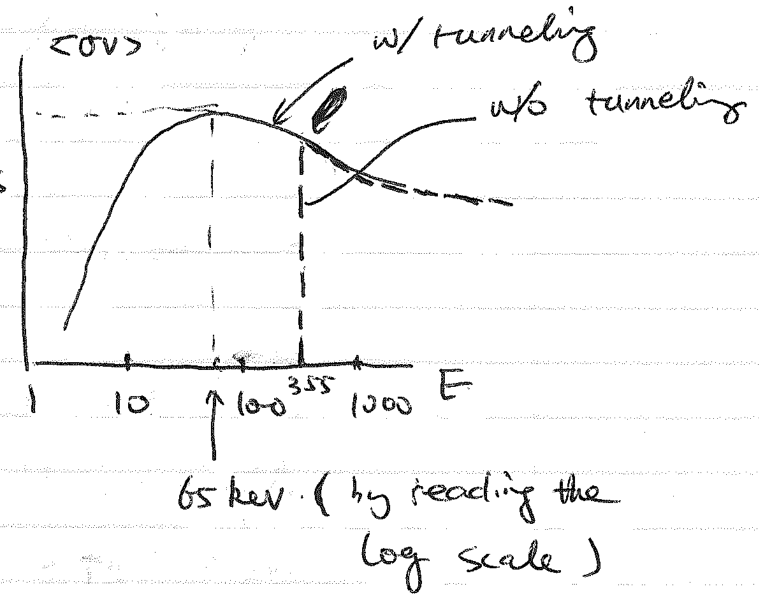
a. Loosely defined as an ionized gas — an electrically neutral medium of positive & negative particles, its size should be ~~set~~ at least several times larger than the Debye length λ_D.

b. The lowest temperature is limited by the requirement of ionization, i.e. the plasma should have enough energy to keep the ionized particles from recombination.

2. a) $\langle \sigma_f \cdot v \rangle$ vs. E

Maximum $\Longrightarrow$ $\sim 10^{21}$ m^3/s

@ $\sim$65 keV.



(b) As shown above.

The ~~coulom~~ Coulomb barrier for dt plasma is about 355 keV, as calculated in your HW # 2.

Without tunneling effect, the D-T plasma must higher energy than 355 keV to have fusions.

As long as the energy is higher than the barrier, the fusion reactivity is just ~~about~~ the same as the one w/ tunneling.

As you can see from the curve, the peak values deceases. (w/o tunneling, peak @ 355 keV)

3. a. Debye shielding distance.

The imposed electrostatic perturbation is attenuated in a plasma. Debye length λ_D is the distance beyond which the perturbation is said to be shielded.

b. $\lambda_D = \sqrt{\dfrac{\varepsilon_0 k T_e}{q_e^2 N}}$ $\qquad \underline{N = 2N_e \quad (\text{if } N_e = N_i)}$

$$= \sqrt{\frac{8.85\times10^{-12} \times 20\times10^{3} \times 1.6\times10^{-19}}{(1.6\times10^{-19})^2 \times 2\times10^{20}}}$$

$$\uparrow$$

$$10^{14}\ cm^{-3} = 10^{20}\ m^{-3}$$

$$= 7.44\times10^{-5}\ m.$$

If 1% Fe ions are involved.

$N_{Fe} = 0.01\, N_{DT}.$ $\longrightarrow$ 26 is the charge of totally stripped Fe.

$N_e = N_{DT} + 26 \times N_{Fe}$

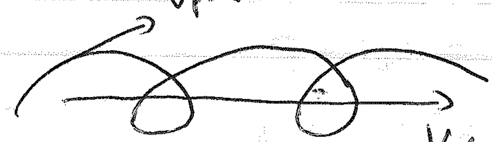
$= 1.26\, N_{DT}.$

$$\lambda_D = \sqrt{\frac{\varepsilon_0 k T_e}{q_e^2 \cdot 2N_e}} = \sqrt{\frac{\varepsilon_0 k T_e}{q_e^2 \cdot (2\times1.26 N_{DT})}}$$

So, λ_D is reduced by a factor of $\sqrt{1.26}$.

4. a)

$V_{particle}$: the real-time velocity of particles.



V_{drift}

↓

The velocity of the guarding center of the particles gyration.

(b) $\vec{V_D} = \dfrac{\vec{E} \times \vec{B}}{B^2}$

if $\vec{E} \perp \vec{B}$, $V_D = \dfrac{E}{B}$

if $\vec{E}$ and $\vec{B}$ have an angle of $45°$,

$\vec{V_D} = \dfrac{\vec{E} \times \vec{B}}{B^2}$ ~~$= \frac{1}{\sqrt{2}} \frac{E}{B}$~~

$V_D = \dfrac{1}{\sqrt{2}} \dfrac{E}{B}$, the direction keeps the same.

5. a.

$$\begin{cases} \nabla \cdot \vec{E} = \dfrac{\rho}{\varepsilon_0} \\ \nabla \cdot \vec{B} = 0 \\ \nabla \times \vec{E} = -\dfrac{\partial \vec{B}}{\partial t} \\ \nabla \times \vec{B} = \mu_0 \vec{J} + \mu_0 \varepsilon_0 \dfrac{\partial \vec{E}}{\partial t} \end{cases}$$

$\rho = q_i n_i + q_e n_e$

$\vec{J} = q_i n_i \vec{V_i} + n_e q_e \vec{V_e}$

⇓

Independent variables

ρ and $\vec{J}$, or $n_i, n_e, \vec{V_i}, \vec{V_e}$.

Or in CGS unit.

$$\begin{cases} \nabla \cdot \vec{E} = 4\pi\sigma \\ \nabla \cdot \vec{B} = 0 \\ \nabla \times \vec{E} = -\dfrac{1}{c} \cdot \dfrac{\partial \vec{B}}{\partial t} \\ \nabla \times \vec{B} = \dfrac{1}{c}\left(4\pi \vec{j} + \dfrac{\partial \vec{E}}{\partial t}\right) \end{cases}$$

b) $D = \varepsilon_0 E = \varepsilon_0 E + P$

$B = \mu H = \mu_0 (H + M)$

In plasma, the electrons and ions are free-moving, so there are no bound charge and bound current. In another word, no polarization P and no magnetization M, so $D = \varepsilon_0 E$ and $B = \mu_0 H$ w/ ε_0 and μ_0 being constant, That's the reason why E and B are used in plasma instead of D and H.

c) $$-\nabla^2 \phi = \frac{q_e N_e + q_i N_i}{\varepsilon_0}$$

$\vec{E} = \nabla \phi$..

$$\Rightarrow \nabla \cdot \vec{E} = -\frac{q_e N_e + q_i N_i}{\varepsilon_0}$$

$$= -\frac{10\% N_i \cdot q_i}{\varepsilon_0}$$

$$= -\frac{0.1 \times 10^{20} \times 1.6 \times 10^{-19}}{8.85 \times 10^{-12}}$$

$$= -1.81 \times 10^{11} \ \frac{V}{m^2}.$$

For one-dimension,

$$\frac{dE}{dx} = -1.81 \times 10^{11} \ \frac{V}{m^2}$$

$$E = -1.81 \times 10^{11} \cdot x \quad \left(\frac{V}{m}\right)$$

↳ that's a huge electric field.

initial radius

6 a). D-T target $R_{b0} = 0.5\,cm$

After compression. $R_b = \sqrt[3]{\frac{1}{1000}} \cdot 0.5 = 0.05\,cm$
$= 5\times10^{-4}\,m.$

Burn time $= \tau_{ic} = R_b \sqrt{\frac{3\bar{m}_i}{10kT_i}}$ $\quad (T_i = 100\,keV)$

$= 5\times10^{-4} \times \sqrt{\frac{3\times\frac{3.3446+5.0085}{2}\times10^{-27}}{10\times100\times10^{3}\times1.6\times10^{-19}}}$

$= 1.40\times10^{-10}\,s.$

Lawson criteria

$\tau \cdot N > 10^{14}\,s\cdot cm^{-3} = \cancel{10^{20}\,s\,m^{-3}}$

$1.4\times10^{-10} \times 10^{25}\,cm^{-3} = 1.4\times10^{15}\,s\cdot cm^{-3} > 10^{14}\,s\cdot cm^{-3}$

So, satisfied.

b). $D+T \rightarrow n+\alpha$ $\quad E_n = 14\,MeV.$

$D+D \rightarrow \begin{cases} p+t \\ n+h \end{cases}$ $\quad E_n = 2.54\,MeV.$

Production rate of D-T neutrons

$= n_D n_T \cdot \langle\sigma v\rangle_{DT}$

production rate of D-D neutrons

$= \frac{1}{2} n_D n_D \langle\sigma v\rangle_{DD,h}$ for $D+D \rightarrow n+h$.

↳ to avoid counting the same possible interaction twice among "indistinguishable" like particles.

$$= \frac{1}{2} n_D^2 \cdot \langle \sigma v \rangle_{DD,n}$$

~~Since~~ Since the $\langle \sigma v \rangle_{DD,p}$ for another branch of DD reaction is about the same, it is ok to write the above rate

$$= \frac{1}{2} n_D^2 \cdot \frac{\langle \sigma v \rangle_{DD}}{2}.$$

Where $\langle \sigma v \rangle_{DD}$ is the ~~total~~ sum of $\langle \sigma v \rangle_{DD,p}$ and $\langle \sigma v \rangle_{DD,n}$

The ratio would be.

$$\frac{R_{DD\,neutron}}{R_{DT\,neutron}} = \frac{\frac{1}{2} n_D^2 \langle \sigma v \rangle_{DD,n}}{n_D n_T \langle \sigma v \rangle_{DDT}} = \frac{1 \langle \sigma v \rangle_{DD,n}}{2 \langle \sigma v \rangle_{DDT}}$$

$$\approx \frac{1}{4} \frac{\langle \sigma v \rangle_{DD}}{\langle \sigma v \rangle_{DDT}}.$$